Econometrics II

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Lecture Structure

1. t-test
2. Testing multiple hypotheses

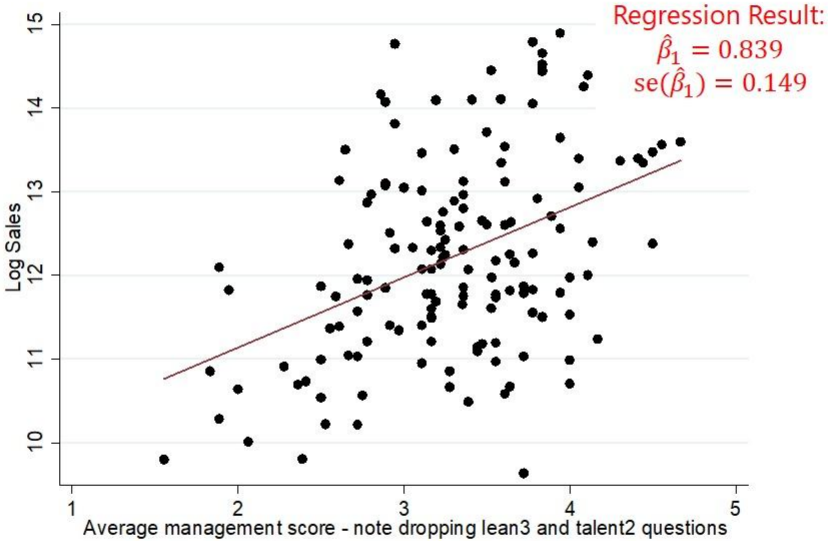
Recap from Last Lecture

- Projection matrix: $\boldsymbol{M_X} = \boldsymbol{I_N} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$
- Estimator for σ^2: $\hat{\sigma}^2 = \frac{RSS}{(N-k)}$
- GM- Theorem: Under GM1-GM4 OLS is BLUE
- t-test: $\frac{\hat{\beta}-\beta}{\sqrt{\hat{\sigma}^2(\boldsymbol{X}'\boldsymbol{X})^{-1}}} \sim t(N-k)$

Example t-Test

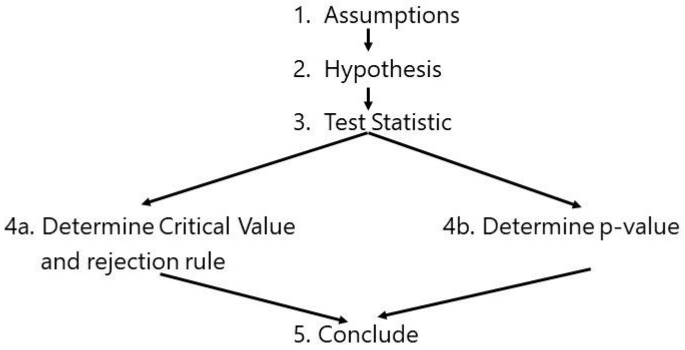
- The following example comes from the World Management Survey conducted by Nick Bloom (Stanford), John Van Reenen (MIT) and coauthors
  - They have developed a questionnaire to measure management quality in firms
  - We relate sales y to the average management score x
  - Focus on German sample (clearest relationship): $N = 148$

Scatter Plot and Regression Line



Perform t-Test

- We can now perform a t-test whether this relationship is significantly different from 0
- Tests always follow the following five steps:



Perform t-Test

1. GM assumptions satisfied
2. Hypotheses: $H_0 : \beta_2 = 0$; $H_0 : \beta_2 \neq 0$
3. Test statistic: $\frac{\hat{\beta}_2 - \beta_2^0}{se(\hat{\beta})} = \frac{(0.839 - 0)}{0.149} = 5.63$ (as we have shown above this is distributed as a t)
4. Critical value from the t-tables (here 148-2 degrees if freedom): $t(100) = 1.984$
5. -> reject H_0

Testing Hypotheses Involving Multiple βs

- We have shown how to test hypotheses about a single β
- In some cases we want to test hypotheses about multiple βs
- Examples of linear hypotheses:
  - H_0: $\beta_2 = 5.2$
  - H_0: $\beta_1 - \beta_3 = 0$
  - H_0: $\beta_1 - \beta_3 = 0 \; and \beta_4 = 6$
  - $H_0 : \beta_j = 0$ for all j

- In matrix notation we can write such hypotheses as follows:

$$H_0 : \underset{r\times k}{R} \underset{k\times 1}{\beta} = \underset{r\times 1}{q}$$

- where r is equal to the number of linear hypotheses

Examples Multiple Hypotheses

- H_0: $\beta_2 = 5.2$: (suppose that we estimated 5 βs)

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} = 5.2$$

- H_0: $\beta_1 - \beta_3 = 0$

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} = 0$$

Examples Multiple Hypotheses

- H_0: $\beta_1 - \beta_3 = 0 \; and \beta_4 = 6$

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

- $H_0 : \beta_j = 0$ for all j

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

- It is important that the hypotheses do not involve redundancies or inconsistencies
- Example for redundancies:

$$H_0: \quad \begin{aligned} 3\beta_2 - 2\beta_1 &= 0 \\ \beta_3 &= 0 \\ -6\beta_2 + 4\beta_1 &= 0 \end{aligned}$$

- Example Inconsistencies:

$$H_0: \quad \begin{aligned} 3\beta_2 - 2\beta_1 &= 0 \\ \beta_3 &= 0 \\ \beta_3 &= 5 \end{aligned}$$

- If one "cleans" the hypothesis of redundancies and inconsistencies: $rank(R) = r$

Test Statistic

- There are 3 equivalent ways of testing such multiple hypotheses
- Here we discuss the procedure that requires the estimation of the unrestricted model and the restricted models (i.e. taken $H_o$for granted and plugging in the restriction e.g. H_0: $\beta_1 - \beta_3 = 0$)
- We then get two *RSS*:
  - RSS_R the sum of squared residuals of the restricted model
  - RSS_U the sum of squared residuals of the unrestricted model
- Which RSS will be higher?
- We can then calculate the following test statistic:

$$\frac{(RSS_R - RSS_U)/r}{RSS_U/N-k} \sim F(r, N-k)$$

F-Distribution